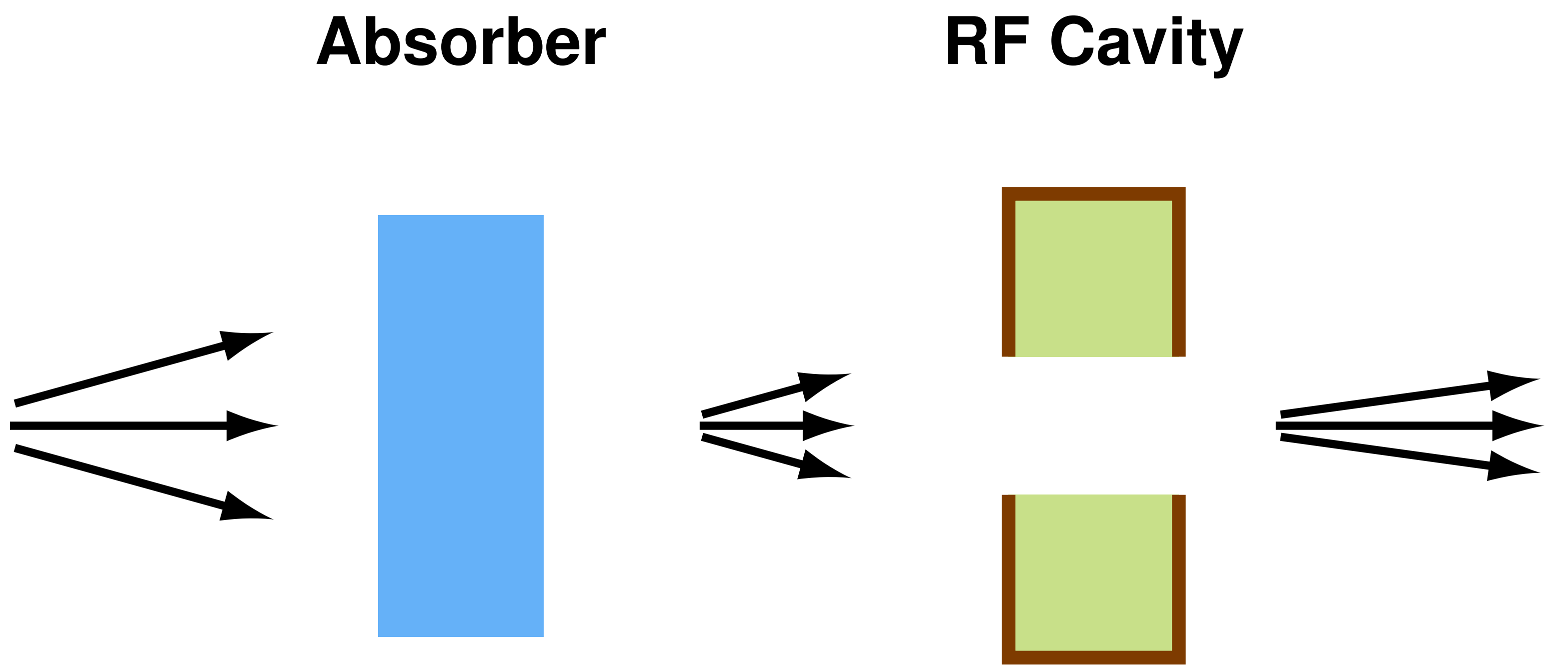
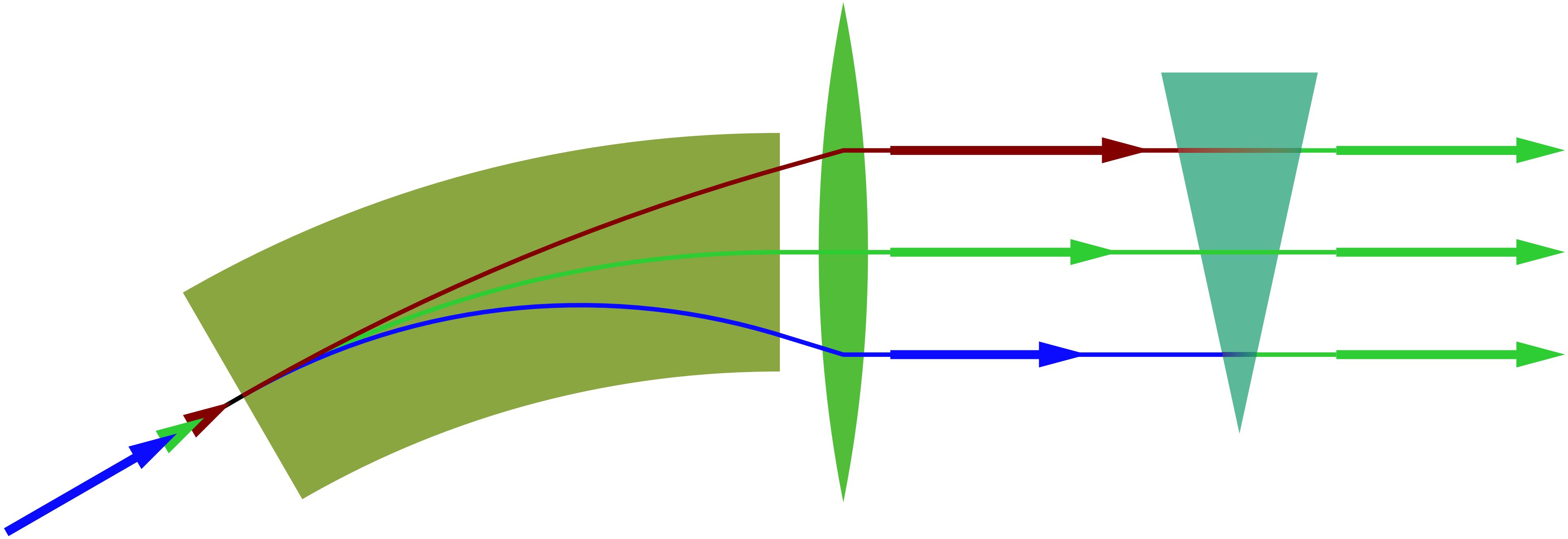
Transverse Cooling

- Particle's momentum reduced, direction same
- Momentum added longitudinally
- Result: transverse momentum reduction, but no effect on longitudinal
- Multiple scattering: low beta function at absorber



Emittance Exchange

- Create dispersion: position depends on energy
- Wedge absorber: energy loss depends on position
- Result: energy spread reduced, but transverse beam size increased
- Effectively cool longitudinal by trading with coolable transverse



Current Development: Rings

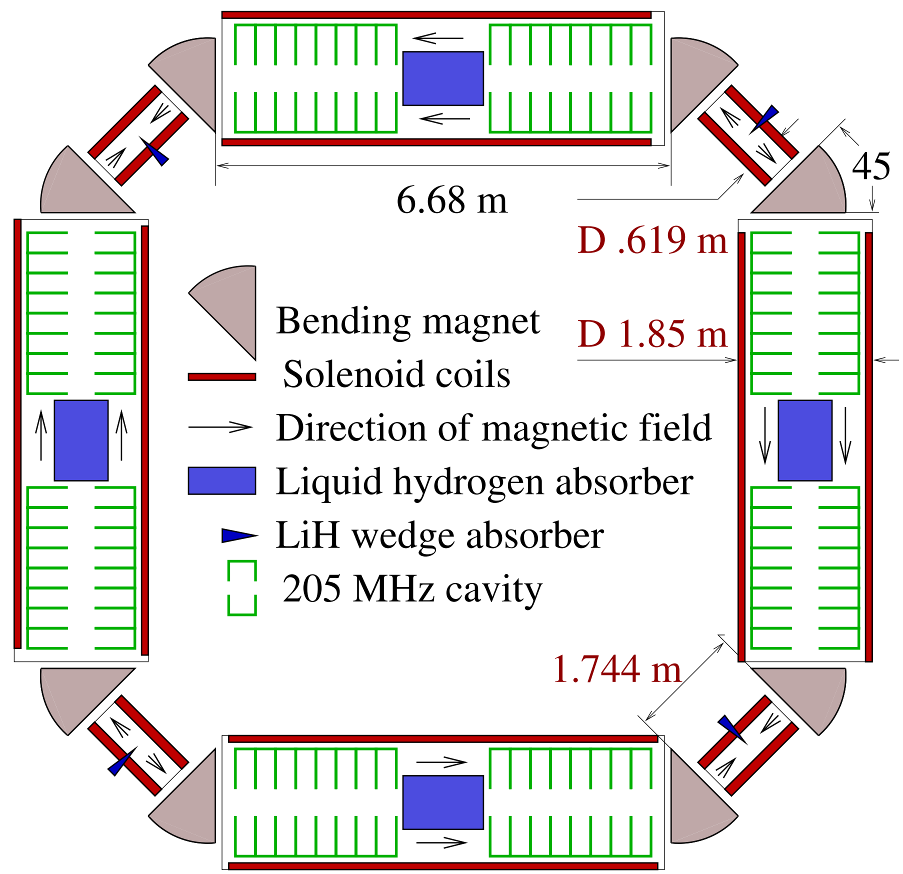
Muon Collaboration

- Properties of rings
  - Give dispersion from bends, allowing longitudinal cooling
  - Multiple passes through same components: lower cost
- Merit factor
  - Transmission times ratio of initial to final emittance (6-D)
  - Factor of increase of central density
  - Has a peak at some number of turns
    - Particles constantly lost
    - Emittance reaches equilibrium

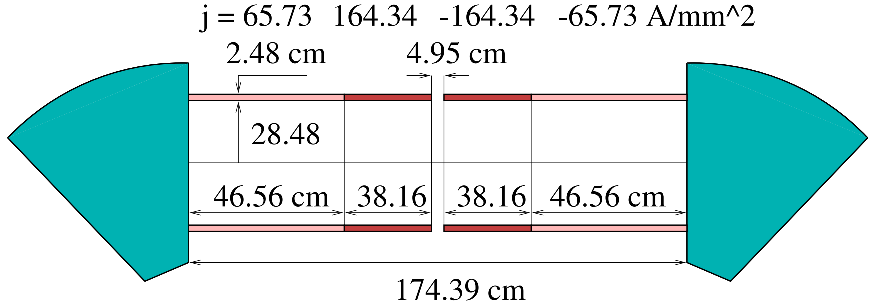
Separated Function Rings
Balbekov

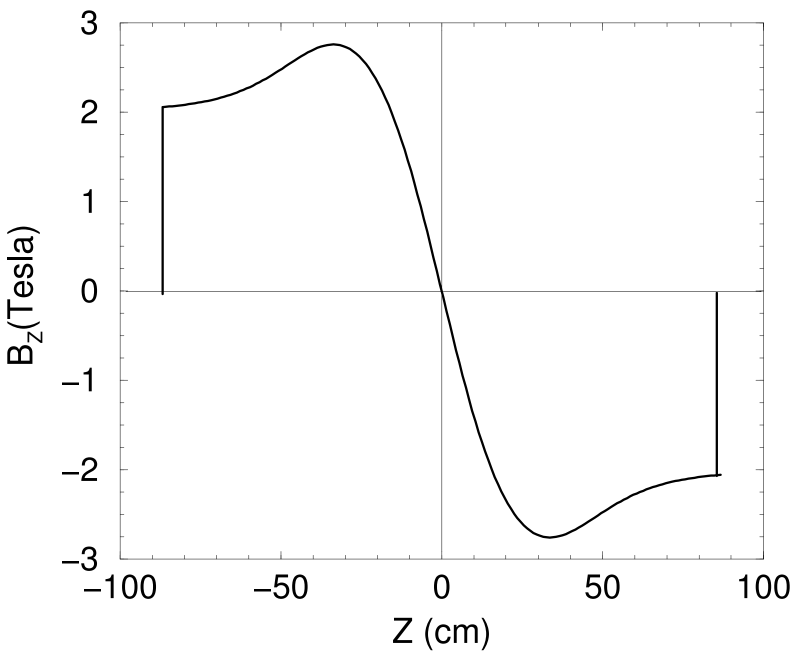
- Transverse cooling sections interleaved with emittance exchange
- Cylindrically symmetric focusing in bends

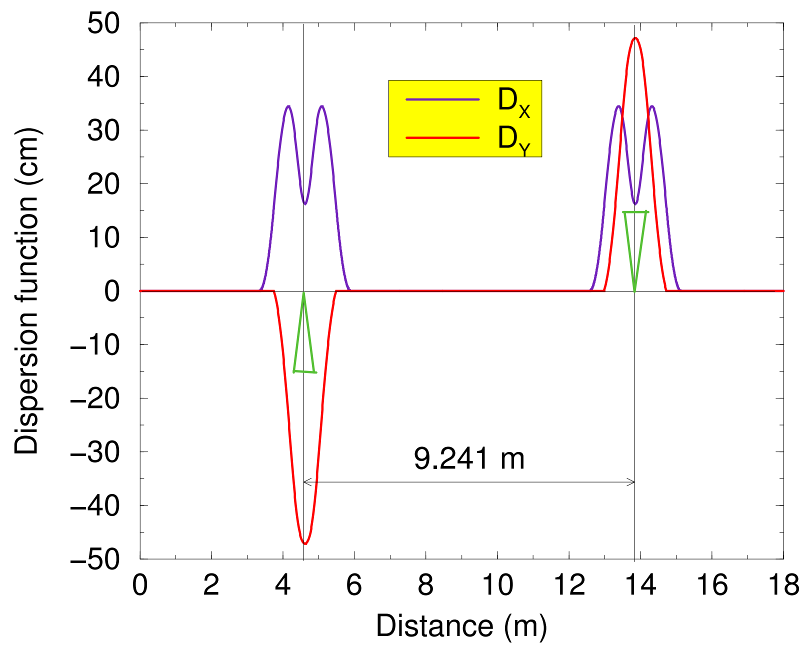
Circumference	36.963 m
Energy	250 MeV
Max B_z	5.155 T
RF Frequency	205.69 MHz
Gradient	15 MV/m



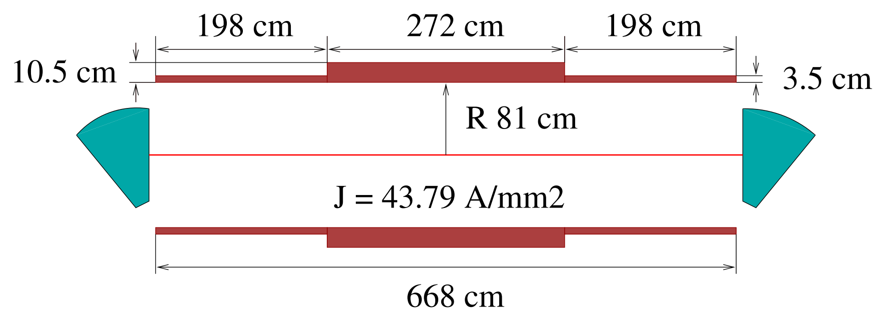
Separated Function Rings
Solenoid Achromats

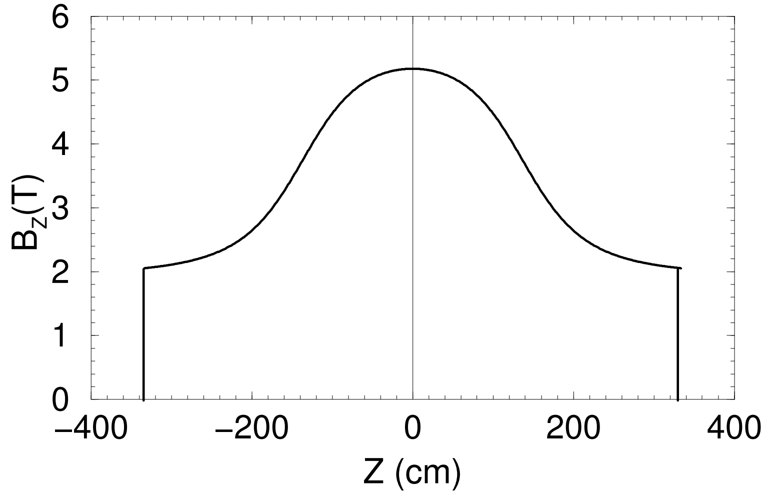


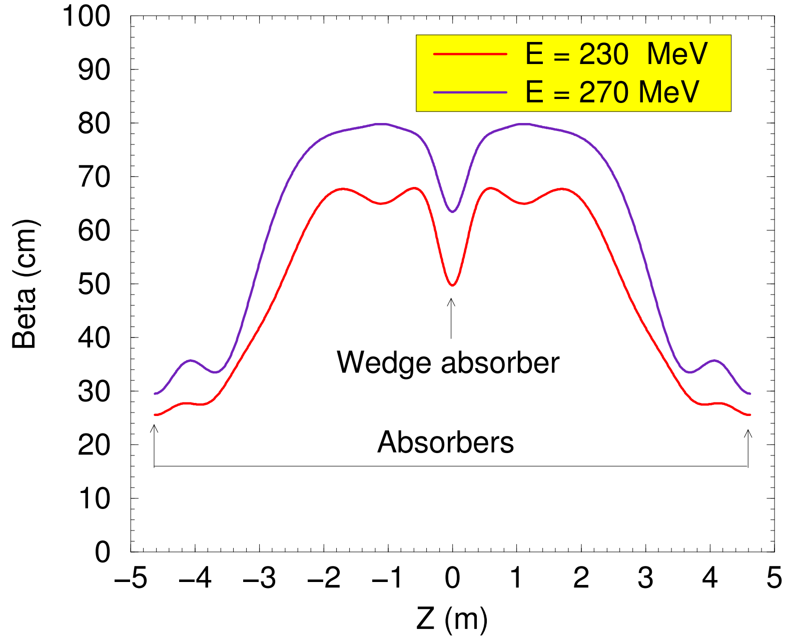




Separated Function Rings
Transverse Cooling Straights

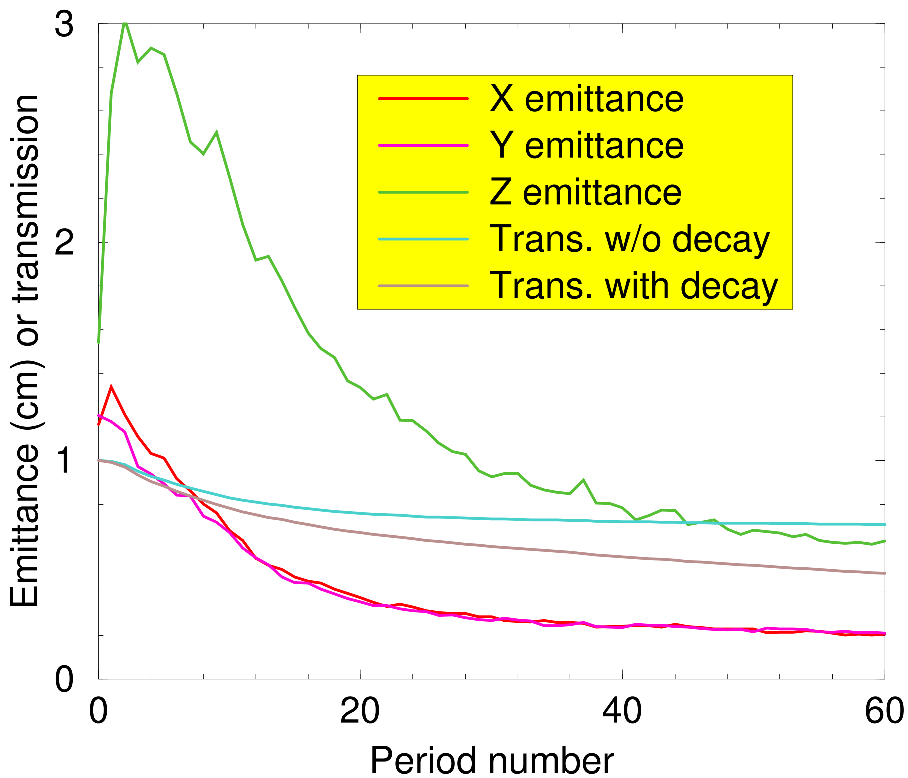






Separated Function Rings Performance

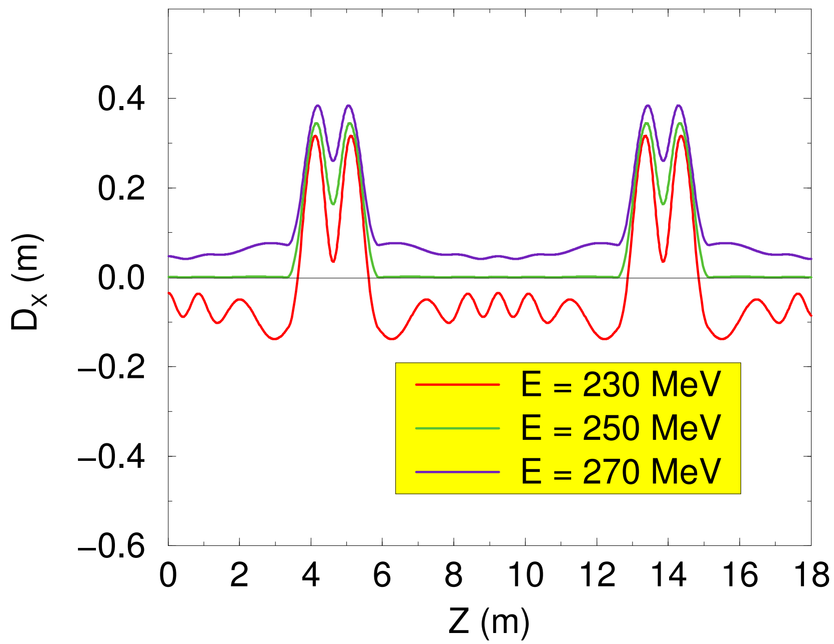
	Before	After
$\epsilon_\perp$ (cm)	1.2	0.21
$\epsilon_\parallel$ (cm)	1.5	0.63
ϵ_6 (cm^3)	2.2	0.028
ϵ_6/ϵ_{60}	1	79
N/N_0, no decay	1	0.71
N/N_0, inc. decay	1	0.48
Merit	1	38



- Nonlinear correlation added to injected beam: uniform v_z (remove transverse momentum)

Separated Function Rings
Nonlinear Dispersion

- Path length function of square of transverse momentum
- Dispersion gives transverse momentum dependence on energy

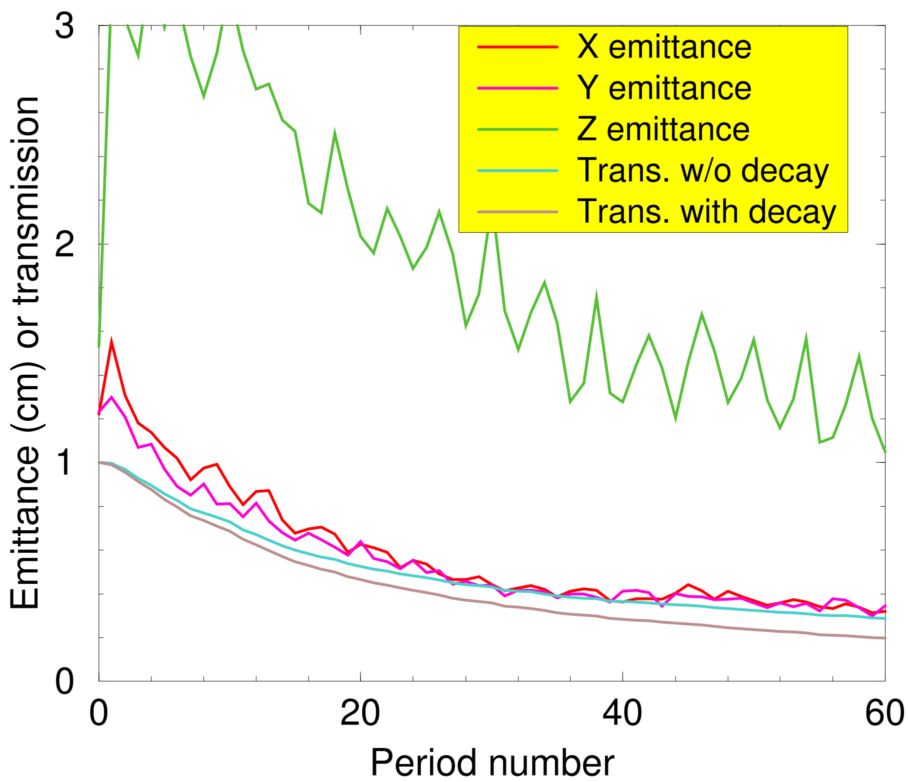


- Large energy spread in beam
- Parametric resonance ($2\nu_s$) with synchrotron oscillations

Separated Function Ring
Remove Cell

- Remove one RF/absorber section to make room for injection/extraction

	Before	After
ϵ_x (cm)	1.2	0.23
ϵ_y (cm)	1.2	0.34
$\epsilon_\parallel$ (cm)	1.5	1.0
ϵ_6 (cm^3)	2.2	0.12
ϵ_6/ϵ_{60}	1	19
N/N_0, no decay	1	0.29
N/N_0, inc. decay	1	0.20
Merit	1	3.9

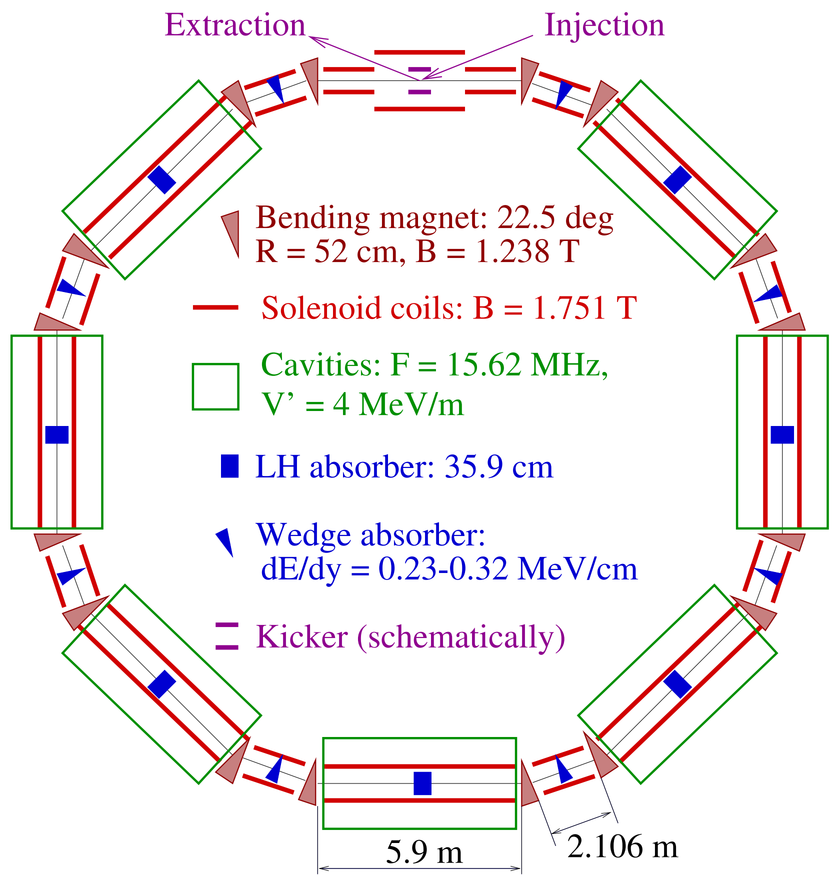


- Problem is longitudinal match: long section lost

Separated Function Ring Bunch Compressor

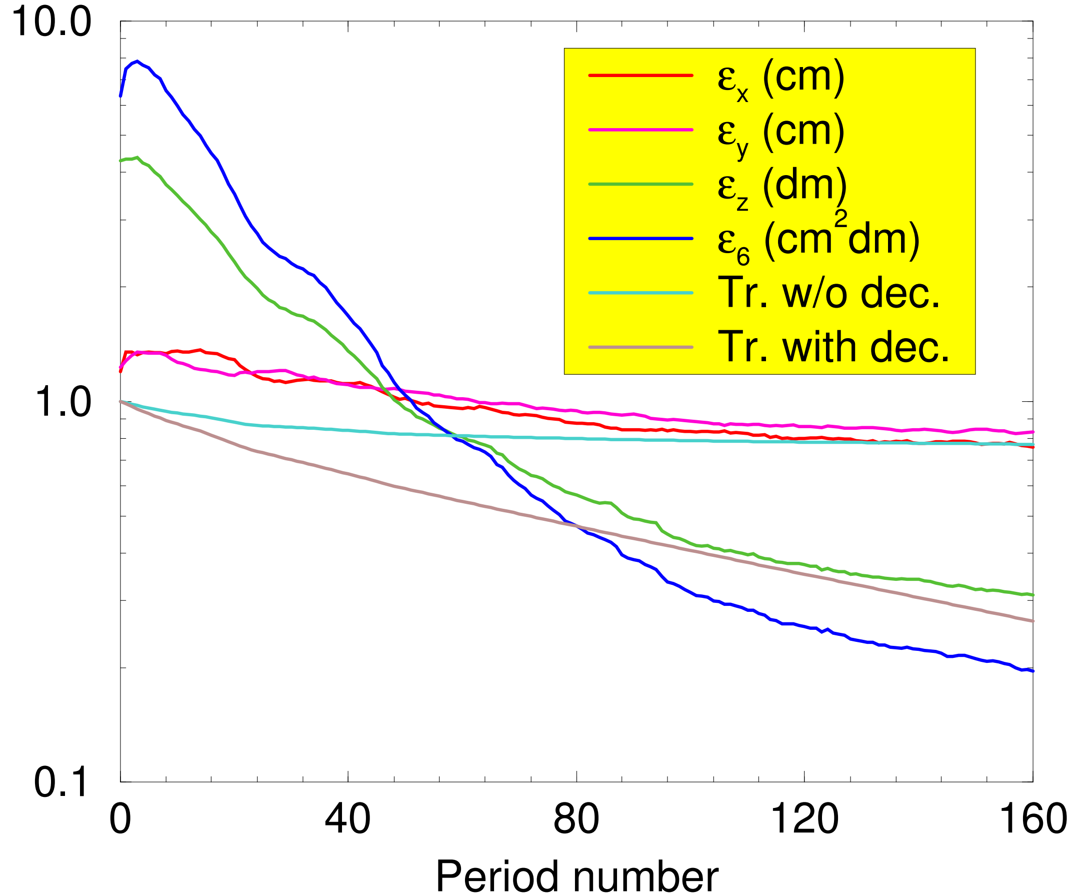
- Goal: reduce extremely large initial longitudinal emittance

Circumference	67.317 m
Energy	220 MeV
Bend field	1.238 T
Max B_z	1.751 T
RF Frequency	15.624 MHz
RF Gradient	4 MV/m



Sep. Fcn. Bunch Compressor Performance

	Before	After
ϵ_x (cm)	1.2	0.76
ϵ_y (cm)	1.2	0.83
$\epsilon_\parallel$ (cm)	43	3.1
ϵ_6 (cm^3)	63	2.0
N/N_0, no decay	1	0.77
N/N_0, inc. decay	1	0.26
Merit	1	8.6

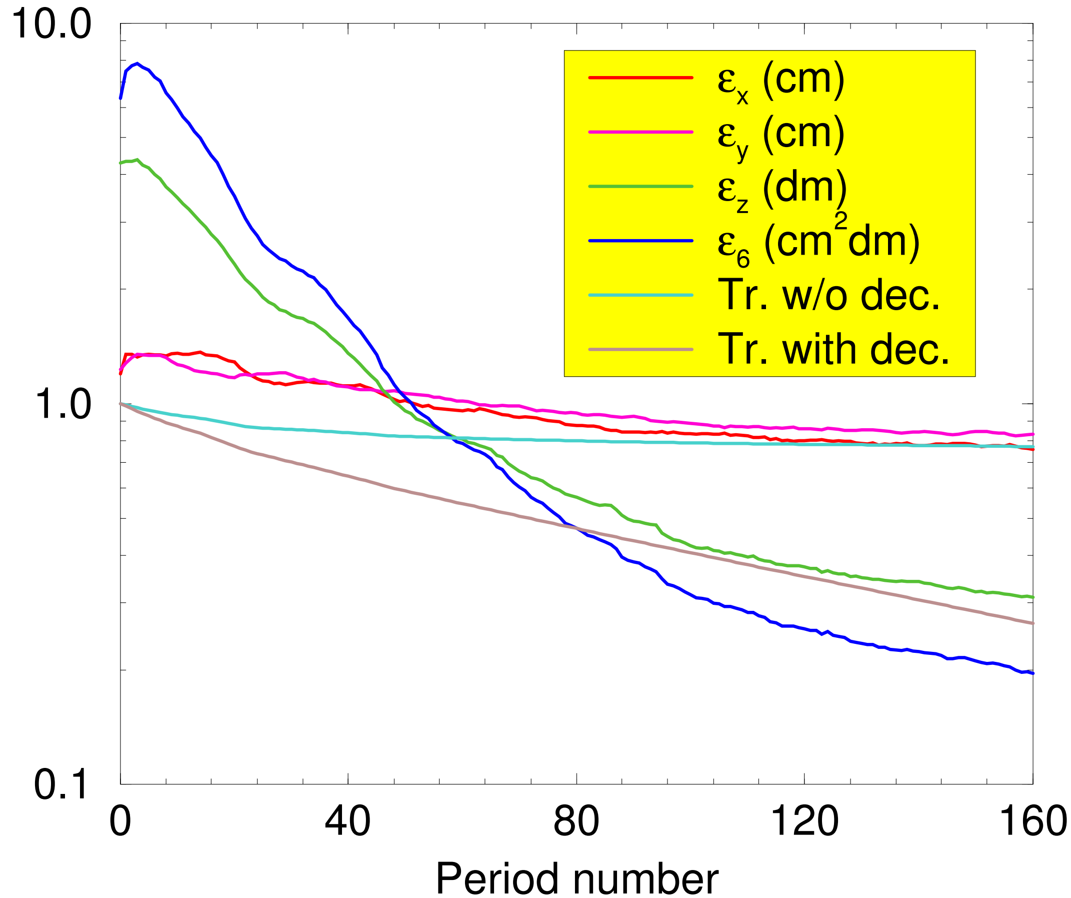


- Lower performance: less absorber (less RF), higher beta function

Sep. Fcn. Bunch Compressor Performance, Empty Straight

- Straight removed for injection

	Before	After
ϵ_x (cm)	1.2	0.83
ϵ_y (cm)	1.2	0.87
$\epsilon_\parallel$ (cm)	43	3.9
ϵ_6 (cm^3)	63	2.8
N/N_0, no decay	1	0.77
N/N_0, inc. decay	1	0.27
Merit	1	5.9



- Lower RF frequency, effect less substantial

Separated Function
Other Comments

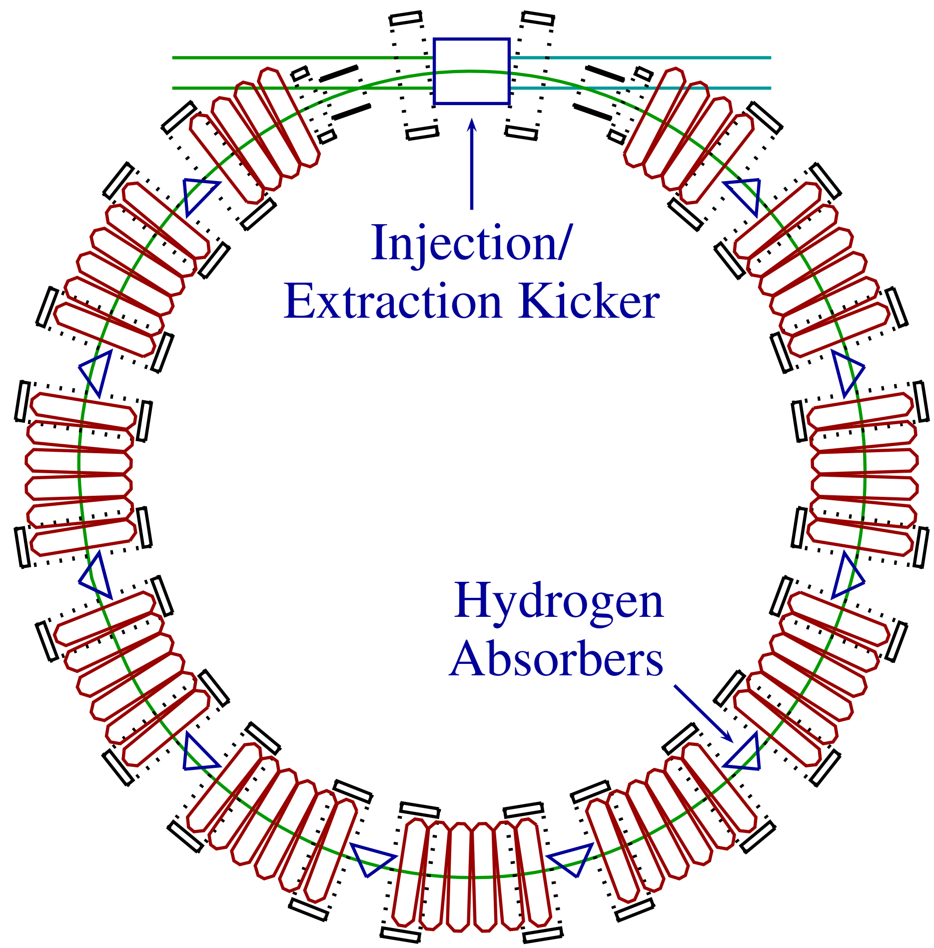
- Long straights, several linear resonances over energy range
  - Large beam, synchrotron oscillations, nonlinearity wash out
- All simulations done with solenoid fields ending abruptly
  - Realistic fields will give nonlinearities
  - Cylindrical focusing symmetry will be broken
- ICOOL simulations show even better performance than Balbekov's
  - Merit factor 94

RFOFO Ring
Palmer *et al.*

- Start with lattice cell from straight cooling channel
  - Compact
    - ⋆ High average accelerating gradient
    - ⋆ Large acceptance (solenoid lattice)
  - Already well optimized for cooling
- Modifications to give longitudinal cooling
  - Bend to generate dispersion
    - ⋆ Dispersion is never removed: dispersion in RF!
    - ⋆ Generate bend by tilting solenoid coils
    - ⋆ Breaks cylindrical symmetry of focusing
  - Put angles on faces of absorber
    - ⋆ Still thick: gives "transverse" cooling
    - ⋆ Gives longitudinal/transverse coupling due to dispersion and angle

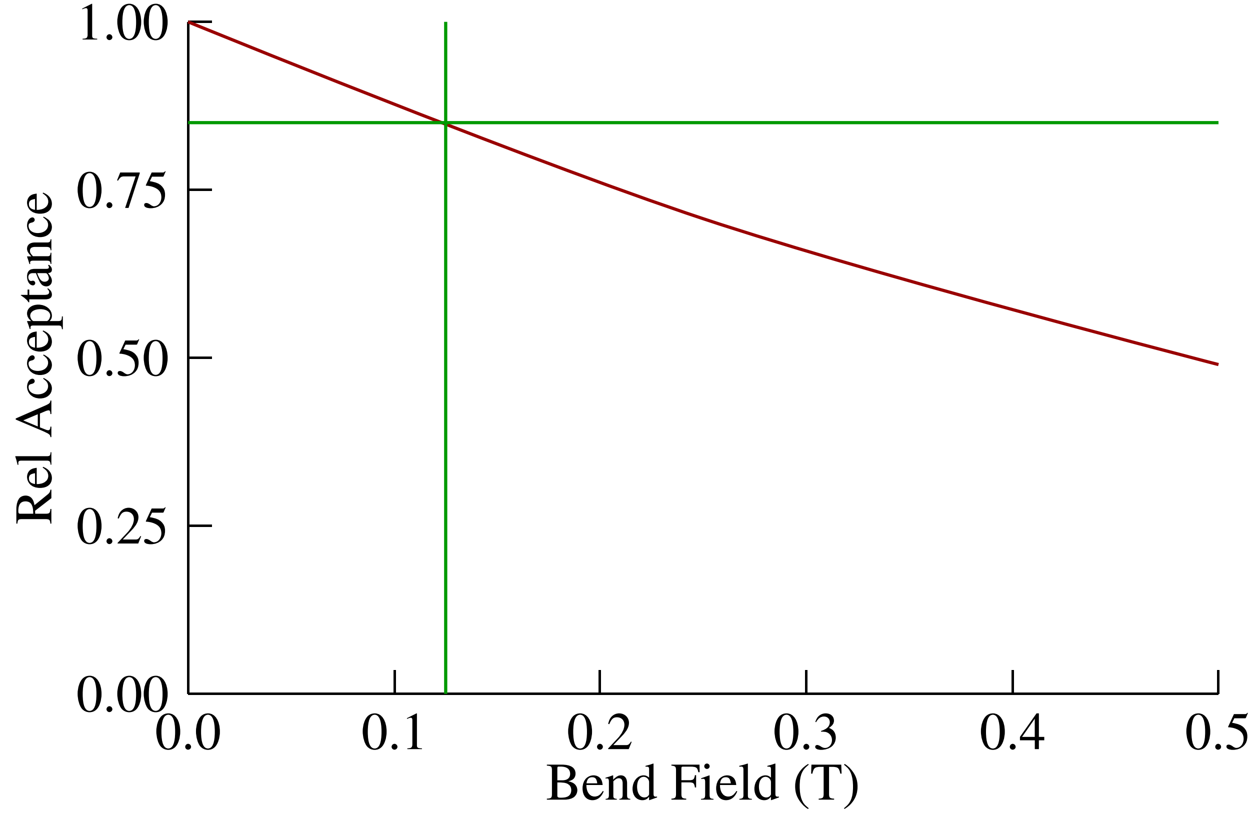
RFOFO Ring Parameters

Circumference	33 m
Momentum	200 MeV/c
Bend Field	0.125 T
Max Solenoid Field	2.7 T
RF Frequency	200 MHz
RF Gradient	12 MV/m

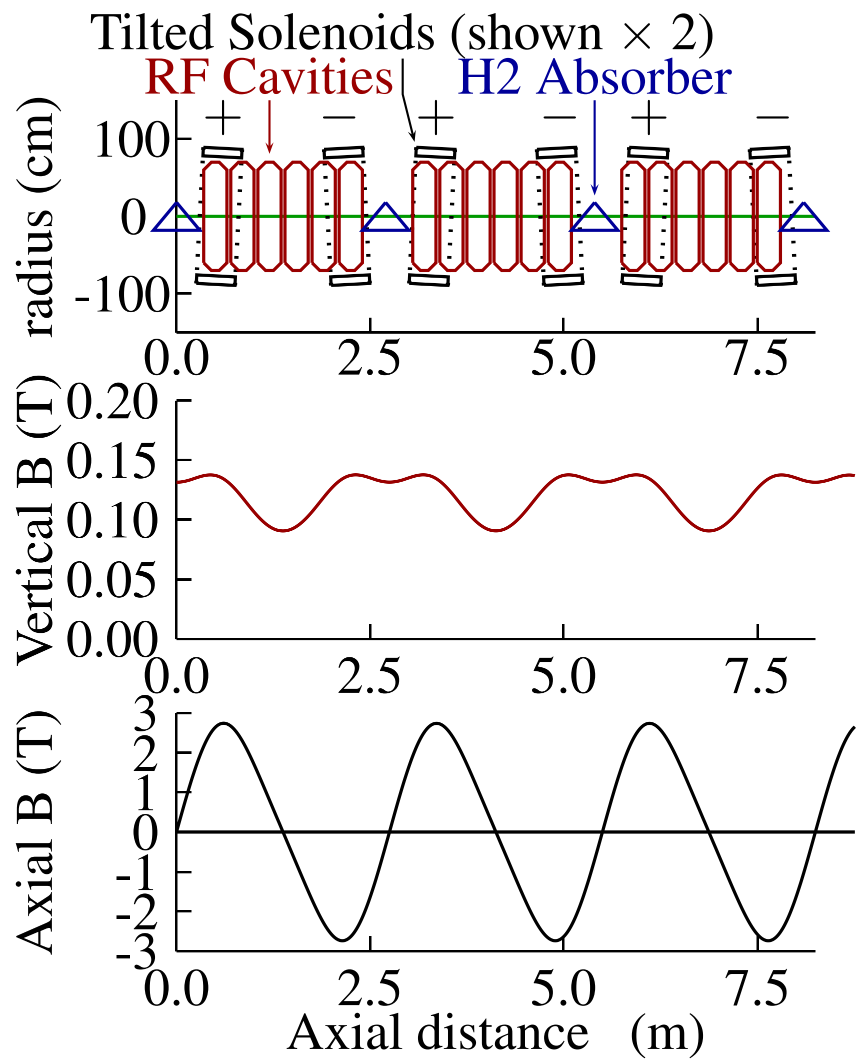


RFOFO Ring
Choice of Bend Field

- Acceptance depends strongly on bend field
- Prefer more bend: lower cost (smaller ring)
- Huge angular acceptance (0.37 rad!!!); needed for performance



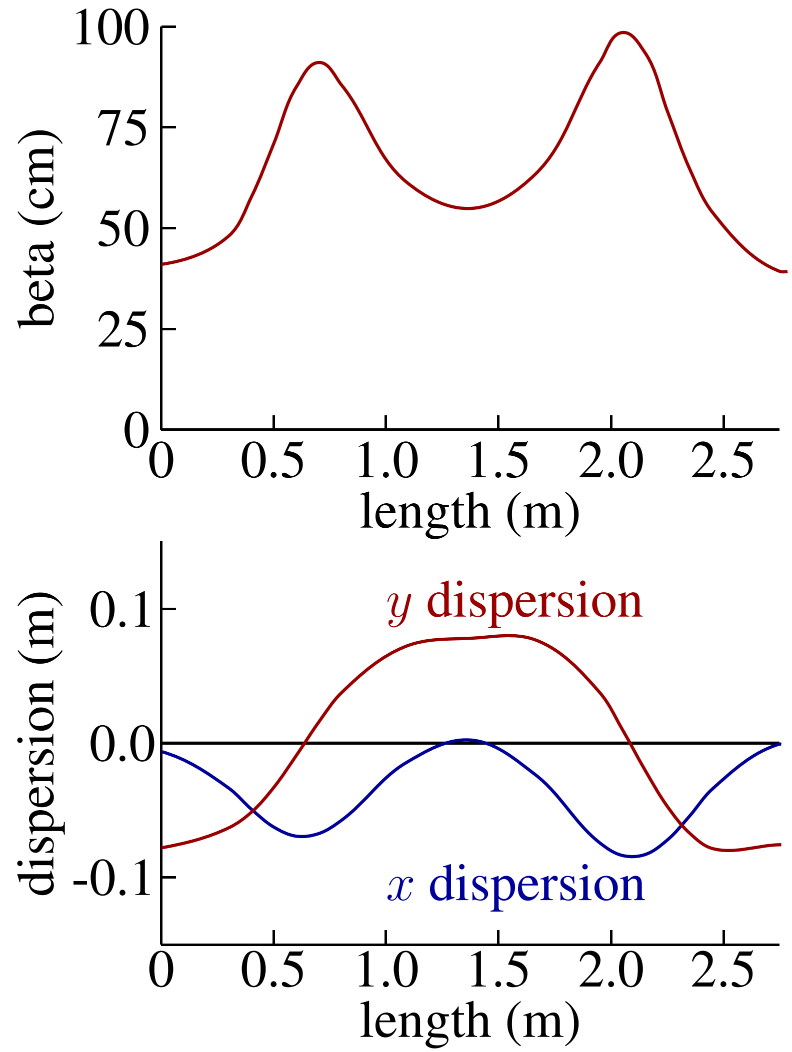
RFOFO Ring
Generating Bend Fields



- Alternating tilts of solenoids produce vertical field
- Not uniform, but close

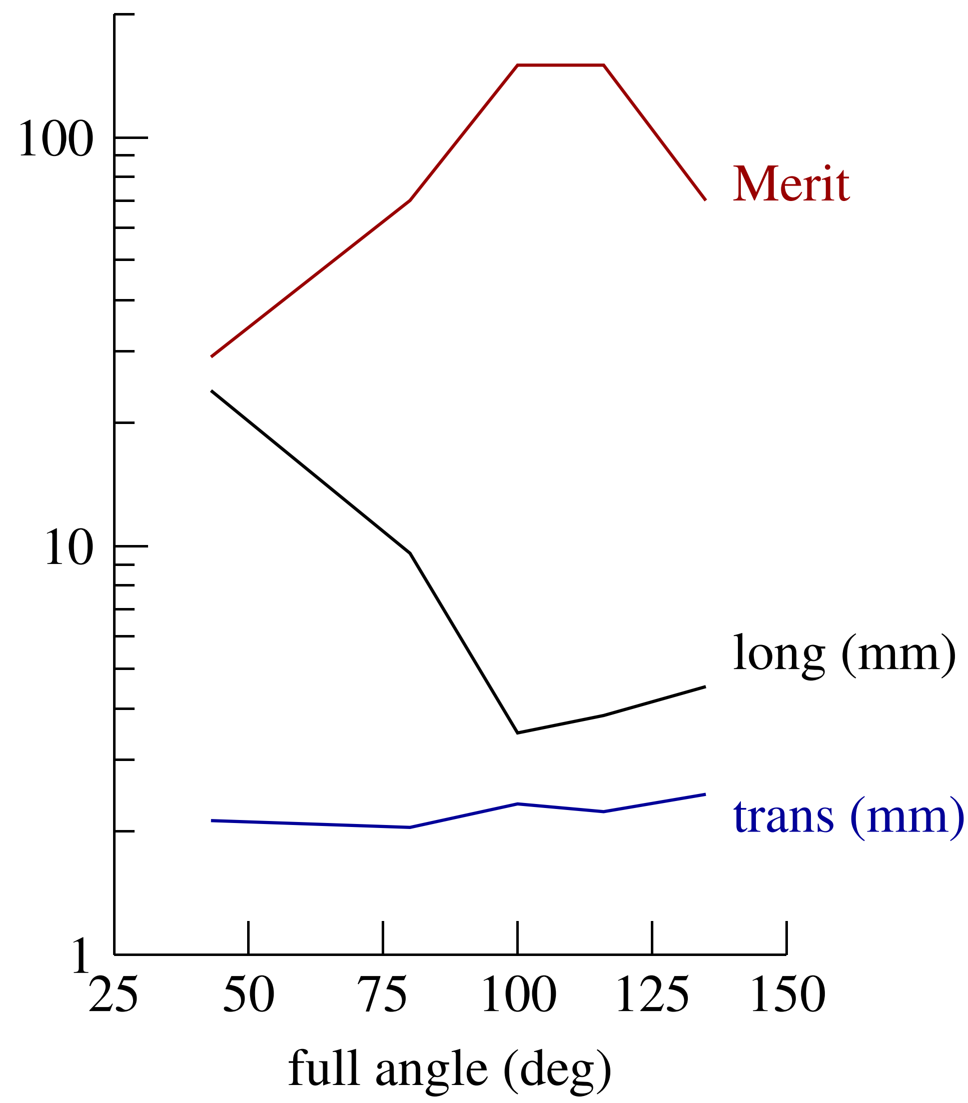
RFOFO Ring
Lattice Functions

- Low beta at absorber
- Maximum dispersion at absorber
- Dispersion rotates back and forth in alternating solenoid field
- Small asymmetry due to energy loss and gain in RF cavities and absorbers
  - Gives horizontal/vertical mixing



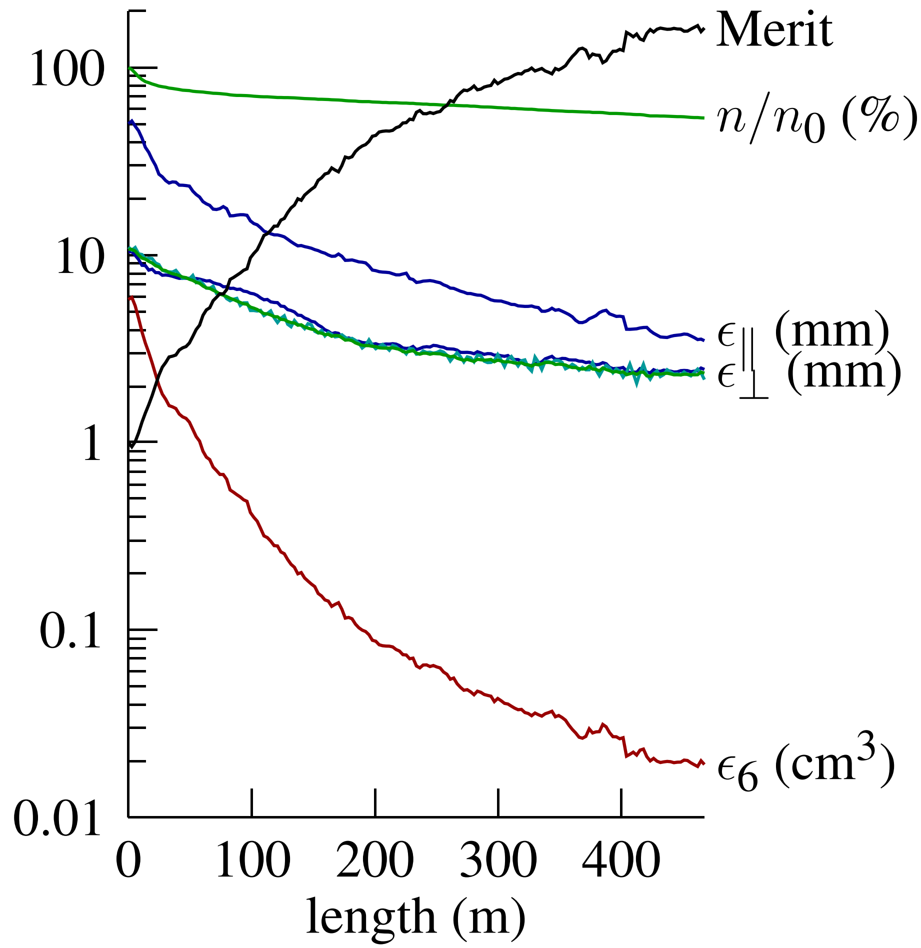
RFOFO Ring
Wedge Angle Optimization

- Maximize merit by varying wedge angle
- Most gain comes from improved longitudinal equilibrium emittance



RFOFO Ring Performance

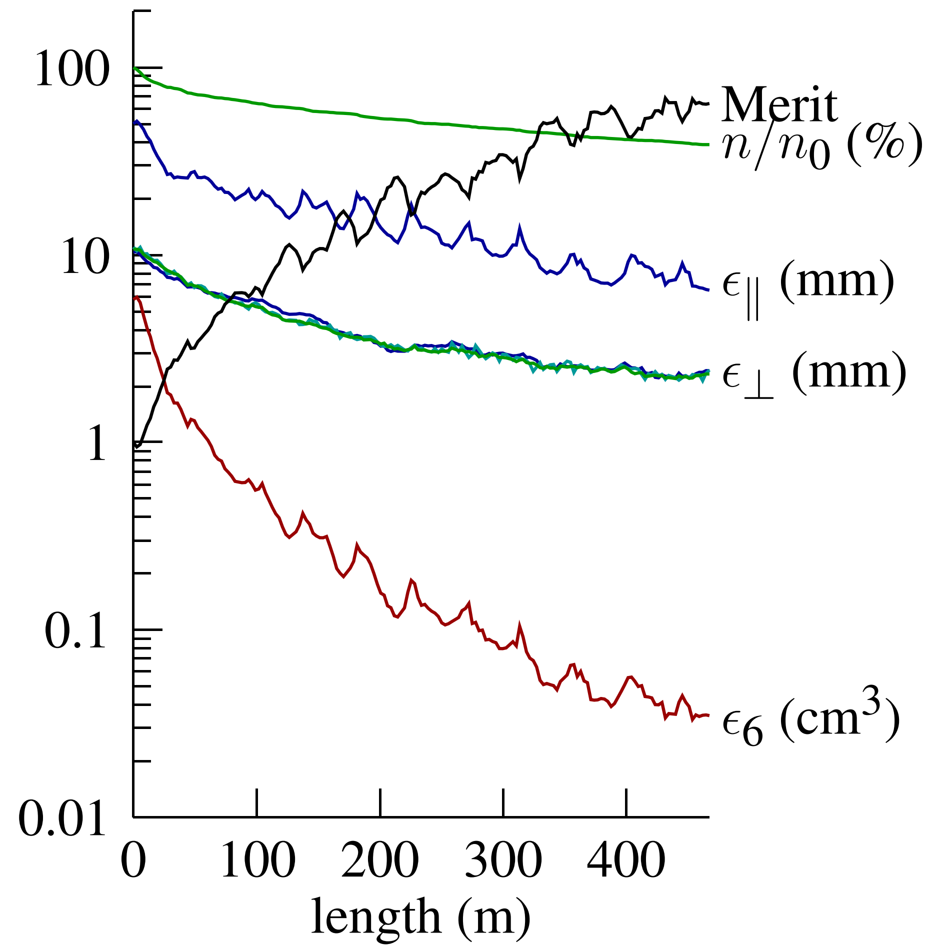
	Before	After
$\epsilon_\perp$ (mm)	10.7	2.3
$\epsilon_\parallel$ (mm)	50.1	3.5
ϵ_6 (cm^3)	5.787	0.019
ϵ_6/ϵ_{60}	1	302
N/N_0, inc. decay	1	0.54
Merit	1	162



RFOFO Ring
Performance with Kicker Gap

	Before	After
$\epsilon_\perp$ (mm)	10.7	2.3
$\epsilon_\parallel$ (mm)	50.1	6.5
ϵ_6 (cm^3)	5.787	0.035
ϵ_6/ϵ_{60}	1	165.7
N/N_0, inc. decay	1	0.39
Merit	1	64

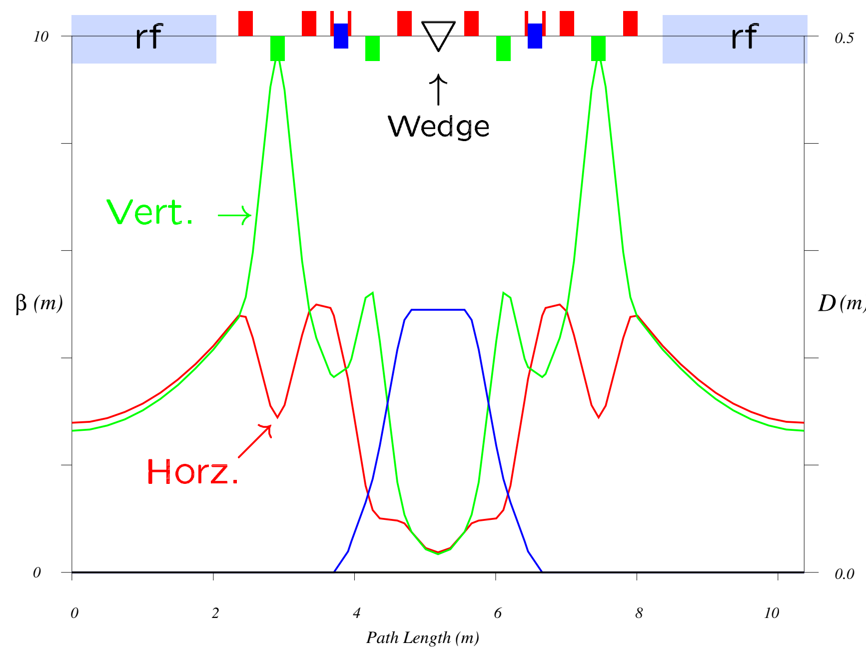
- Problem: longitudinal match
- More losses also (related)



Quadrupole Ring
Garren, Kirk

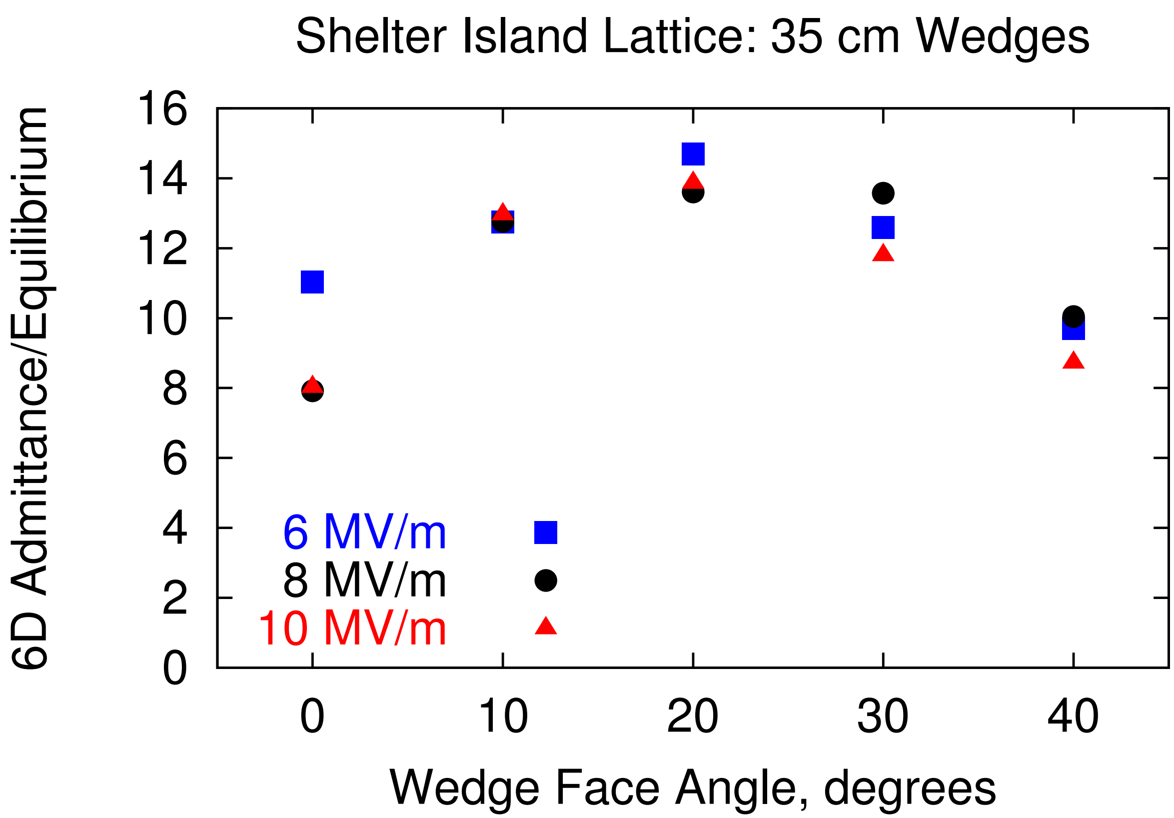
- Motivation
  - Easier to design lattice (dispersion suppression, etc.)
  - More experience than with solenoids
  - Injection and extraction potentially easier
- Thick wedge: both cooling and longitudinal/transverse coupling

Circumference	165 m
Momentum	500 MeV/c
Magnet length	20 cm
Magnet aperture (full)	40 cm
Space between magnets	25 cm
Max pole tip field	4.2 T
RF Frequency	200 MHz
RF Gradient	8 MV/m



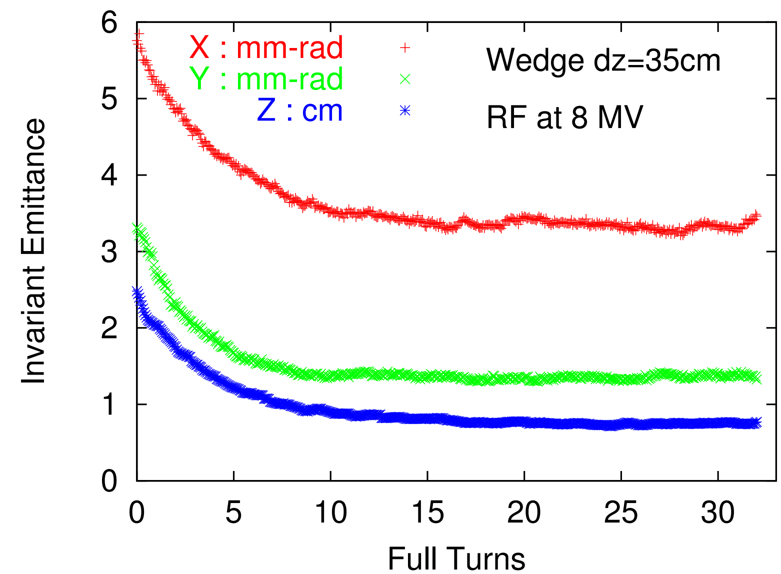
Quadrupole Ring
Wedge Angle Optimization

- Admittance: emittance of the largest beam that can be transmitted in lattice
- Vary wedge angle to maximize ratio of admittance to equilibrium emittance

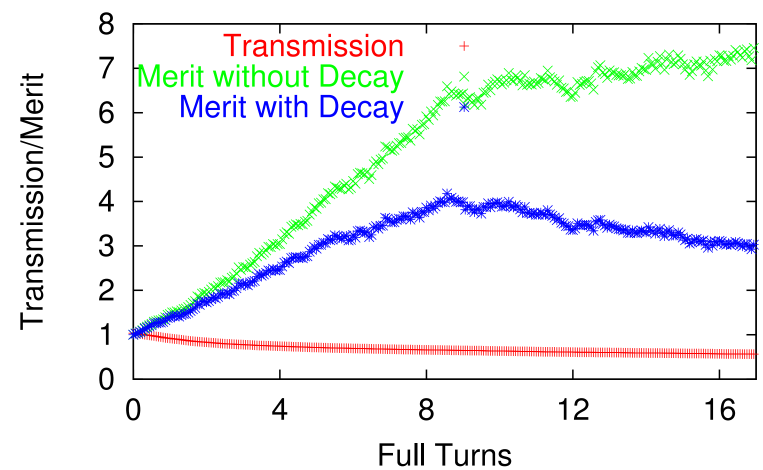


Quadrupole Ring Performance

	Before	After
ϵ_x (mm)	5.76	3.64
ϵ_y (mm)	3.31	1.43
$\epsilon_{\parallel}$ (mm)	24.8	9.8
ϵ_6 (mm^3)	473	51.0
ϵ_6/ϵ_{60}	1	9.27
N/N_0, no decay	1	0.64
N/N_0, inc. decay	1	0.42
Merit	1	3.9



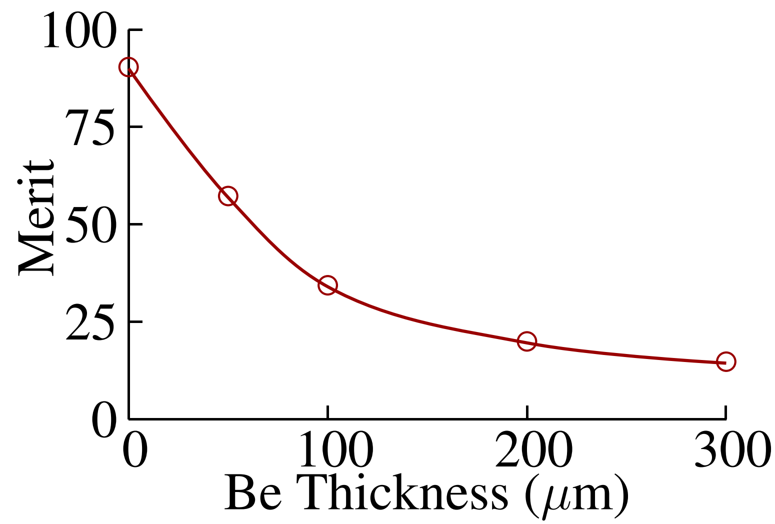
Shelter Island Lattice: 20 degree Wedges



- Performance limitations
  - Limited acceptance
  - Low real-estate gradient
- Equilibrium emittance similar to other systems

General Problems
Foils and Windows

- Multiple scattering in windows and foils reduces performance
  - Higher Z materials
  - Generally at higher beta functions
- Example: system with merit 92, windows on hydrogen container
  - Conventional Al windows, 0.5 mm: merit 31
  - Very thin Al windows, 0.125 mm: merit 61
    - Need to redesign containment
- Example: vary thickness of Be RF windows
  - Poor performance for conventional thickness
  - Thin windows not so bad
    - Possible at liquid nitrogen temperature
  - Eliminate windows
    - Loss of RF gradient



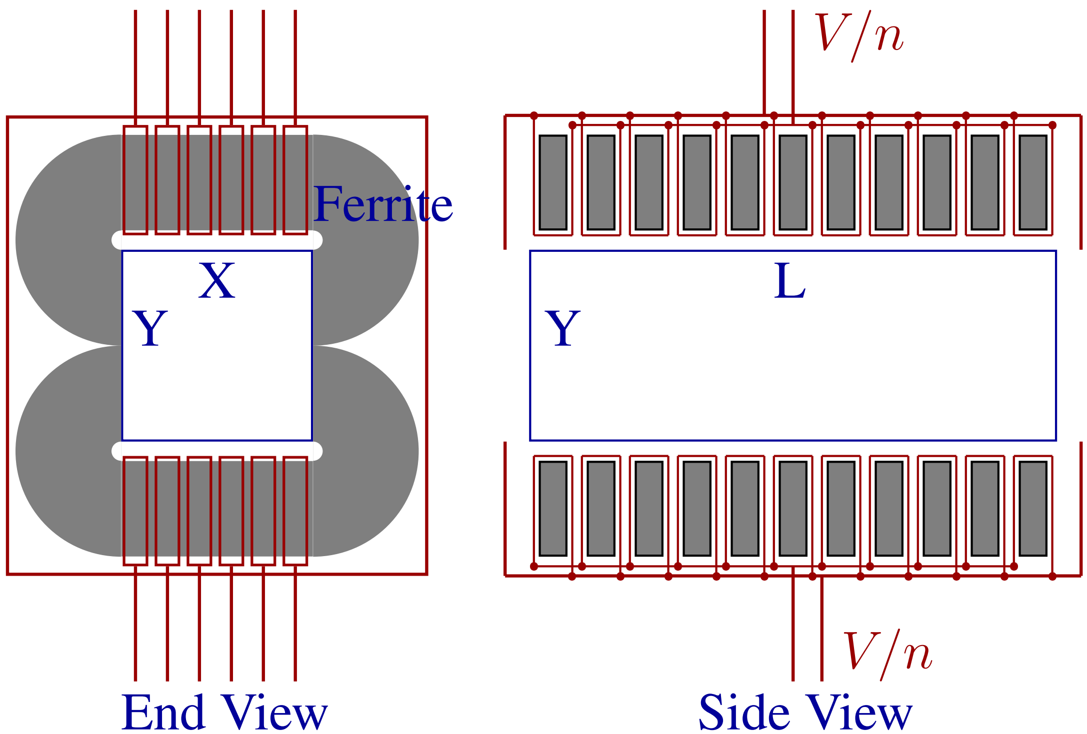
General Problems
Injection/Extraction

- Difficulty leaving space for kicker: longitudinal matching
- Extreme kicker requirements

	μ Ring	CERN $\bar{p}$
$B \cdot L$ (T-m)	0.30	0.088
Length (m)	1	5
Field (T)	0.3	0.018
Rise time (ns)	50	90
x aperture (cm)	42	8
y aperture (cm)	63	25
Voltage (kV)	3970	800
Stored energy (J)	10,450	13

- Far beyond state of the art for kickers
- Similar to induction linac; borrow techniques from there

Injection/Extraction Kicker Magnet



- Drive and subdivide flux return (reduce voltage)
- Conducting box removes stray field return
- Can remove ferrite ($\cos\theta$ configuration): double stored energy, current, same voltage
  - No limitation on rise time

Injection/Extraction Magamp Supply

- Non-resonant system
  - Need separate drivers for injection and extraction
  - Example, need 48 magamps (about $20 M?)
- Resonant
  - Both injection and extraction
    - Pulses in opposite direction
    - Could add switch in low-current section of magamp
      - Also allows lengthening pulse separation
  - More efficient (twice?)
  - Same example, only 12 magamps

Cooling Quality Definition

- Define quality as

$$Q = \frac{d\epsilon_6}{dN}\frac{N}{\epsilon_6}$$

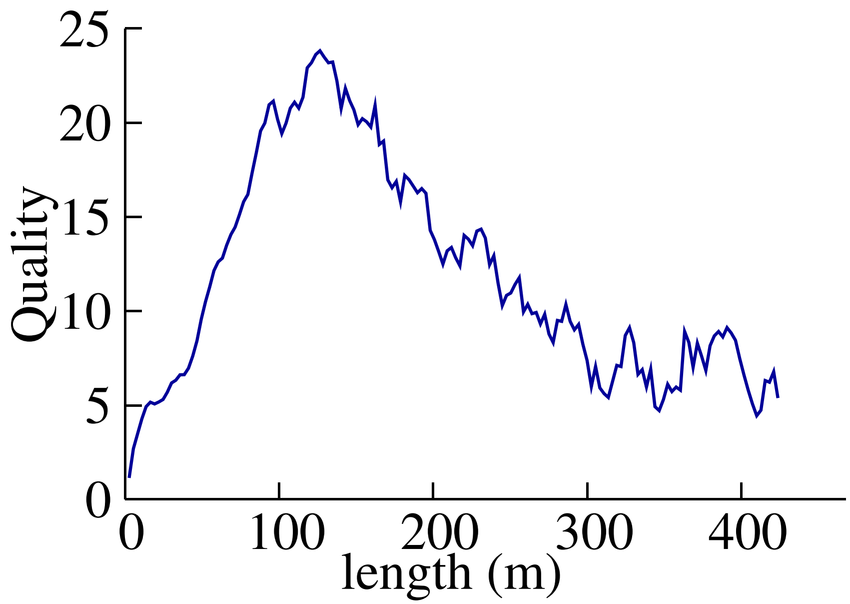
  - Constant Q

$$\frac{N}{N_0} = \left(\frac{\epsilon_6}{\epsilon_{60}}\right)^{1/Q}$$

  - Collider parameters: ϵ_6 reduced by 10^6, only half of particles lost: $Q = 20$

Cooling Quality Evolution in Ring

- Initially: scraping/dynamic losses and mismatch, low Q
- As approach equilibrium emittance cooling rate decreases, low Q
- When losses are only from decays and far from equilibrium, peak Q
- Need peak far above desired value to get average desired
- Could maintain peak by tapering system, but no longer ring: expensive!



Cooling Quality
Theoretical Approximation

- Assume far from equilibrium emittance
- Ignore contribution from longitudinal growth in absorbers (small error)

$$Q \approx \frac{2c\tau}{\beta mc^2}\frac{\Delta E}{L}$$

- ΔE is energy gained in length L
- Only two parts lattice dependent
  - Velocity (β), but very weak dependence
  - $\Delta E/L$: average real-estate energy gain/loss
    - Energy lost must be restored by RF: $V\cos\phi/L$
    - More RF gradient will improve Q
    - Also, being closer to crest will improve
- To approximate approach to equilibrium, multiply by $(\epsilon_6^{1/3} - \epsilon_{6,\mathrm{eq}}^{1/3})/\epsilon_6^{1/3}$

Theoretical Development
Kim, Wang

- Linear theories for 6D cooling developed for
  - Quadrupole lattices
  - Solenoid lattices, requiring cylindrically symmetric linear focusing in bends
    - Equations for dispersion in both planes
    - 5 invariants in this system!
  - Assumes dispersion removed in RF cavities
- Include damping and stochastic effects
  - Predict equilibrium emittances